

Determine if $\int_0^1 \frac{\sec^2 x}{x\sqrt{x}} dx$ converges or diverges. Justify your answer properly.

SCORE: ____ / 15 PTS

$$0 \leq \frac{1}{x\sqrt{x}} \leq \frac{\sec^2 x}{x\sqrt{x}}$$

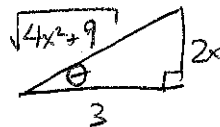
$$\hookrightarrow \int_0^1 \frac{1}{x^{\frac{3}{2}}} dx \text{ DIVERGES (} p = \frac{3}{2} > 1 \text{)}$$

$$\text{so } \int_0^1 \frac{\sec^2 x}{x\sqrt{x}} dx \text{ DIVERGES}$$

Find $\int x^2 \sqrt{4x^2 + 9} dx$.

SCORE: ____ / 30 PTS

$$x = \frac{3}{2} \tan \theta \longrightarrow \tan \theta = \frac{2x}{3}$$
$$dx = \frac{3}{2} \sec^2 \theta d\theta$$



$$\int \left(\frac{9}{4} \tan^2 \theta\right) (3 \sec \theta) \left(\frac{3}{2} \sec^2 \theta\right) d\theta$$
$$= \frac{81}{8} \int \tan^2 \theta \sec^3 \theta d\theta$$
$$= \frac{81}{8} \int (\sec^5 \theta - \sec^3 \theta) d\theta$$
$$= \frac{81}{8} \left[\frac{1}{4} \sec^3 \theta \tan \theta + \frac{3}{4} \int \sec^3 \theta d\theta - \int \sec^3 \theta d\theta \right]$$
$$= \frac{81}{8} \left[\frac{1}{4} \sec^3 \theta \tan \theta - \frac{1}{4} \left(\frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \ln |\sec \theta + \tan \theta| \right) \right] + C$$
$$= \frac{81}{32} \sec^3 \theta \tan \theta - \frac{81}{64} \sec \theta \tan \theta - \frac{81}{64} \ln |\sec \theta + \tan \theta| + C$$
$$= \frac{81}{32} \left(\frac{\sqrt{4x^2+9}}{3}\right)^3 \left(\frac{2x}{3}\right) - \frac{81}{64} \left(\frac{\sqrt{4x^2+9}}{3}\right) \left(\frac{2x}{3}\right) - \frac{81}{64} \ln \left| \frac{\sqrt{4x^2+9}}{3} + \frac{2x}{3} \right| + C$$
$$= \frac{1}{16} x (\sqrt{4x^2+9})^3 - \frac{9}{32} x \sqrt{4x^2+9} - \frac{81}{64} \ln |\sqrt{4x^2+9} + 2x| + C$$

$$\int_0^{\infty} \frac{1}{x(\ln x)^3} dx \quad \text{DISCONT @ } x=0, 1$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int \frac{1}{u^3} du = -\frac{1}{2u^2}$$

$$= -\frac{1}{2(\ln x)^2}$$

$$= \int_0^{\frac{1}{e}} \frac{1}{x(\ln x)^3} dx + \int_{\frac{1}{e}}^1 \frac{1}{x(\ln x)^3} dx$$

$$+ \int_1^e \frac{1}{x(\ln x)^3} dx + \int_e^{\infty} \frac{1}{x(\ln x)^3} dx$$

$$= \lim_{N \rightarrow 1^+} \int_N^e \frac{1}{x(\ln x)^3} dx$$

$$= \lim_{N \rightarrow 1^+} \left. -\frac{1}{2(\ln x)^2} \right|_N^e$$

$$= \lim_{N \rightarrow 1^+} \left(-\frac{1}{2(\ln N)^2} + \frac{1}{2} \right)$$

$$\ln N \rightarrow 0$$

$$= -\infty$$


DIVERGES

$$\int_{-1}^0 \frac{e^{\frac{1}{x}}}{x^2} dx \quad \text{DISCONT @ } x=0$$

$$u = \frac{1}{x}$$

$$du = -\frac{1}{x^2} dx$$

$$\int -e^u du = -e^u \\ = -e^{\frac{1}{x}}$$


$$= \lim_{N \rightarrow 0^-} \int_{-1}^N \frac{e^{\frac{1}{x}}}{x^2} dx$$

$$= \lim_{N \rightarrow 0^-} -e^{\frac{1}{x}} \Big|_{-1}^N$$

$$= \lim_{N \rightarrow 0^-} (e^{\frac{1}{N}} + e^{-1})$$

$$\frac{1}{N} \rightarrow -\infty$$

$$= e^{-1}$$

$$\int \sec^2 \sqrt{x} dx$$

$$u = \sqrt{x}$$

$$x = u^2$$

$$dx = 2u du$$

$$= \int 2u \sec^2 u du$$

$$\begin{array}{l} 2u \int \sec^2 u \\ 2 \int \tan u \\ 0 \int -\ln|\cos u| \end{array}$$

$$= 2u \tan u + 2|\ln|\cos u|| + C$$

$$= 2\sqrt{x} \tan \sqrt{x} + 2|\ln|\cos \sqrt{x}|| + C$$

$$\int \frac{8x-48}{x^4-16} dx \quad \text{HINT: Do NOT factor the numerator}$$

$$\frac{8x-48}{(x+2)(x-2)(x^2+4)} = \frac{A}{x+2} + \frac{B}{x-2} + \frac{C(2x)+D(2)}{x^2+4}$$

$$\begin{aligned} 8x-48 &= A(x-2)(x^2+4) \\ &+ B(x+2)(x^2+4) \\ &+ C(2x)(x+2)(x-2) \\ &+ D(2)(x+2)(x-2) \end{aligned}$$

$$x=2: -32 = B(4)(8) \rightarrow B = -1$$

$$x=-2: -64 = A(-4)(8) \rightarrow A = 2$$

$$x=0: -48 = A(-2)(4) + B(2)(4) + D(2)(2)(-2)$$

$$-48 = -16 - 8 - 8D \rightarrow D = 3$$

$$\text{COEF OF } x^3: 0 = A + B + 2C$$

$$0 = 2 - 1 + 2C \rightarrow C = -\frac{1}{2}$$

$$\text{SANITY CHECK: LHS} = \frac{-24}{(5)(13)} = \frac{-24}{65}$$

$$x=3$$

$$\text{RHS} = \frac{2}{5} - \frac{1}{1} + \frac{-\frac{1}{2}(6)+3(2)}{13}$$

$$= -\frac{3}{5} + \frac{3}{13}$$

$$= \frac{-39+15}{65}$$

$$= \frac{-24}{65} \checkmark$$

$$\int \left(\frac{2}{x+2} - \frac{1}{x-2} + \frac{-\frac{1}{2}(2x)+3(2)}{x^2+4} \right) dx$$

$$= 2 \ln|x+2| - \ln|x-2| - \frac{1}{2} \ln(x^2+4) + 3 \tan^{-1} \frac{x}{2} + C$$